

Power spot price models with negative prices

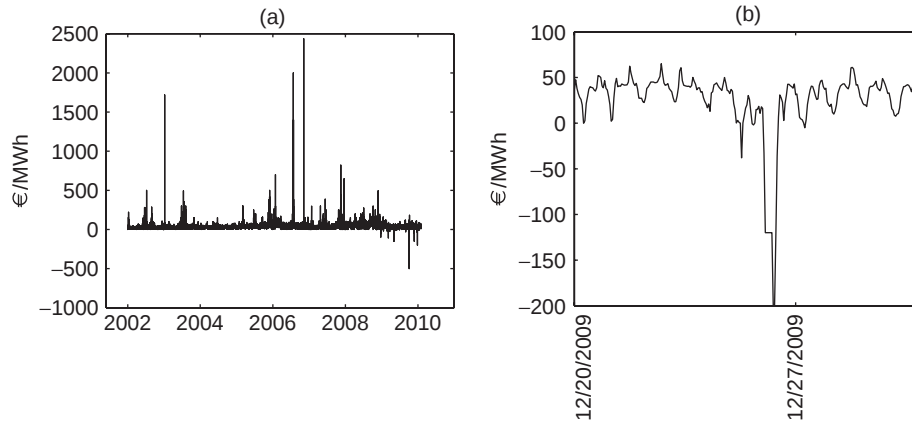
Stefan Schneider

E.ON Energy Trading SE, Holzstrasse 6, 40221 Düsseldorf, Germany;
email: stefan.schneider@eon.com

Negative prices for electricity are a recent development in European power markets. Negative hourly prices have been permitted at Germany's European Energy Exchange (EEX) spot market since autumn 2008 and have occurred frequently since, reaching values as low as €−500/MWh. However, in some non-European markets such as those in the US, Australia and Canada, negative prices have been observed over longer periods. Negative prices are, in fact, natural in electricity spot trading. Plant flexibility is limited and costly, and, therefore, incurring a negative price for an hour can actually be economically optimal overall. Negative prices pose a basic problem for stochastic price modeling in that going from prices to log prices is not possible. So far, this has been dealt with using “work-arounds”. In this paper, a thorough approach is advocated, based on the area hyperbolic sine transformation. The transformation is applied to spot modeling of the German EEX and the Electric Reliability Council of Texas's West Texas (ERCOT West) market, and an exemplary valuation of an option is carried out. It is concluded that the area hyperbolic sine transform is a natural starting point for modeling negative power prices. It can be integrated into common stochastic price models without adding much complexity. Moreover, this transformation might, in general, be more appropriate for power prices than the log transformation, considering the fundamentals of power price formation. Negative prices can significantly affect a business and a thorough treatment is therefore indispensable.

1 INTRODUCTION

The German power exchange, the European Energy Exchange (EEX), began to permit negative price outcomes for the spot auction in autumn 2008. Since then, negative prices have occurred frequently, reaching values as low as €−500/MWh (see Figure 1 on the next page). This has triggered considerable attention and debate, not only in the energy trading business but also in the media. Paying money instead of demanding it when selling a commodity to a counterparty appears counterintuitive. However, this is a natural consequence of the properties of the commodity power. Generally, this is because the generation of power has little flexibility due to technical and regulatory constraints. When it is not possible for a generation facility to follow a demand

FIGURE 1 The history of EEX hourly prices.

(a) EEX hourly prices. (b) EEX hourly prices, Christmas 2009.

slump closely, its generation is sold off at a discounted price, sometimes even at negative prices.

For the modeler, negative prices pose a basic problem: the initial transition from prices to log prices, the default for modeling stochastic price processes, is not possible. Surprisingly (to the best of the author's knowledge), no solution that realistically deals with this issue has been proposed so far. Instead, the existing approaches are work-arounds. The simplest solution is to exclude all negative occurrences from analysis, or to introduce a shifted price with zero level at the observed minimum price (eg, $\text{€}-500/\text{MWh}$) (see Sewalt and De Jong (2003) and Knittel and Roberts (2001)). The justification given for using work-arounds is that negative prices are supposed to have little influence because they are relatively rare. However, energy traders and business practitioners do not feel comfortable with this (see Sprenger and Laege (2009)). On the contrary, they notice a significant impact of negative prices on the value of their position when holding a structured off-peak position, for example.¹ Looking again at the EEX price history for 2009 (Figure 1), it indeed appears justified to consider negative or downward jumps as having replaced the upward spikes in the times before the economic downturn.

In this paper we develop a simple but effective way of dealing realistically with negative (and positive) spot prices. This is achieved by replacing the log transformation with the area hyperbolic sine transformation. An appealing feature of this approach

¹ A profile with hourly varying loads in the low-demand periods.

is its practicability with regard to stochastic price modeling. Basic cases can still be expressed in closed form. Moreover, there are general indications that the area hyperbolic sine transformation is better suited for power prices, regardless of the occurrence of negative prices.

2 NEGATIVE POWER PRICES

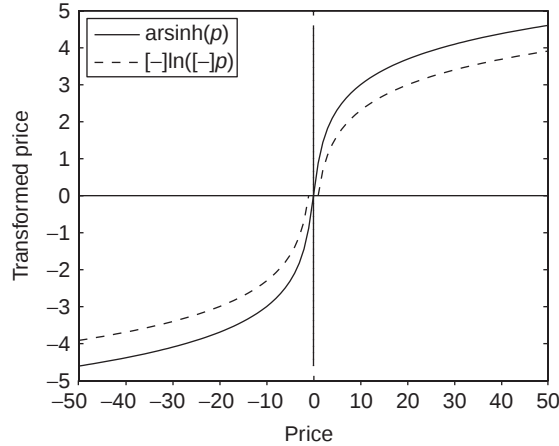
In this section, a brief account of the economic and technical background of negative power spot prices is given. For a detailed overview of this subject see, for example, Dettmer and Jacob (2009).

The German EEX spot exchange was the first market in Europe to permit negative prices. The Scandinavian Nord Pool spot followed by the end of 2009. However, outside Europe, power markets permitting negative prices are well-established. The possibility of negative prices was built into their design from the beginning. Two striking examples are the Electric Reliability Council of Texas's (ERCOT) West Texas market and the Australian Energy Market Operator (AEMO) in South Australia. They exhibit very frequent (and often strongly) negative prices.

Negative prices at EEX occur when very low demand coincides with high supply. Typically, low-demand situations occur, for example, on public holidays and Sunday nights, amplified by slumps in industrial activity due to the economic crisis. A typical high-supply situation is constituted by a high wind power infeed. The grid operator, who brings the electricity produced from wind generators to the market, then bids this power at EEX at a negative price to achieve market clearing. Also, production from conventional power plants can be offered on the market for negative prices. This occurs, for example, when the plant produces above the marginal generation costs for matching the total demand in a given hour. It could then nevertheless be economically optimal to run the plant for that hour if the loss from the negative price is smaller than the costs for a modification of the production schedule would be. An alternative production schedule causes costs due to a number of factors, including ramping costs, start-up costs and costs for procuring energy from alternative sources during a mandatory downtime of the plant. Another negative price situation is constituted by grid transmission bottlenecks (for example, high wind production in Denmark cannot be sufficiently transported to hydro pump storages in northern Scandinavia).

To sum up, negative prices occur because production of power has limited flexibility for economic, technical and regulatory reasons. When it is not possible for a power generation facility to follow a demand slump closely, its generation is sold off at a discounted price. This is why it was appropriate to allow for negative prices at EEX.² It

² Previously, spot auctions at EEX frequently attained no intersection of bid and offer curves, necessitating partial market clearing only.

FIGURE 2 Area hyperbolic sine transformation.

Price transformation $\sinh^{-1}(p)$ along with $\ln(p)$ and $-\ln(-p)$ for comparison. "arsinh" stands for area hyperbolic sine.

was shown that doing so is economically rational in order to optimize market clearing (see Viehmann and Sämisch (2009)).

It remains to be seen whether other markets will follow the model of EEX and Nord Pool, especially considering the fact that European power markets are becoming increasingly integrated. Interestingly, for another commodity, natural gas (delivered to the National Balancing Point), there have been some negative prices in the past. The situation was analogous to power: a production facility (a gas field) provided excess production during a low-demand period that could not be sufficiently adapted for technical reasons.

3 MODELING CONCEPT

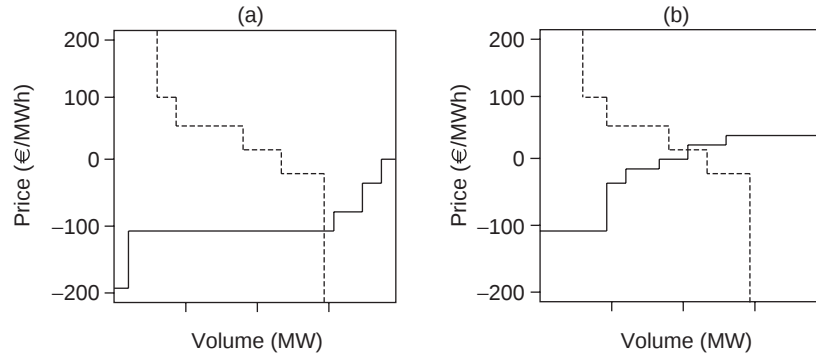
3.1 The area hyperbolic sine transformation

Stochastic modeling of power prices usually starts with log transforming $x = \ln p$ of the original prices p . Instead, we propose to replace the log transformation by the area hyperbolic sine transformation:

$$x = \sinh^{-1} \left(\frac{p - \xi}{\lambda} \right) \quad (3.1)$$

where ξ is an offset and λ is a scale parameter. The behavior of this function compared with the natural logarithm is depicted in Figure 2. The most important property is the

FIGURE 3 Bid/demand (dashed line) and ask/supply (solid line) curves of hour 1 (part (a)) and hour 14 (part (b)) for the EEX auctions of December 26, 2009 (schematically redrawn).



asymptotic log behavior:

$$\sinh^{-1}(p) = \ln(p + \sqrt{p^2 + 1}) \approx \operatorname{sgn}(p) \ln(2|p|)$$

for $|p| \rightarrow \infty$.³ The log function is already a good approximation for small $|p| > 2$. The positive and negative log-like parts are connected by an approximately linear part at $|p| \approx 0$.

The transformation appears to be a natural choice because it preserves the log behavior, which is a proven method. However, it is now presupposed that the properties of prices $p < 0$ are a “mirror image” of $p > 0$. We recall a basic rationale for the log transformation. The volatility/variability of price increases with the absolute price level $dp \sim p$. This is why returns $dp/p(t)$ or log returns $\ln p(t) - \ln p(t-1)$ are studied. It appears plausible that this is also the case for negative power prices $|dp| \sim |p|$ for $p < 0$. This can be substantiated by studying bid/ask curves of EEX spot auctions (see Figure 3). As is well-known, the curves get steeper when moving to high prices, and thus the spot price becomes more variable/spiky when the auction outcome (intersection of curves) moves into that price region. We see that the same holds for the negative price region. Note that this finding already rules out the approach of shifting the zero price level down to a negative lowest level.

³ The offset and scale parameter are temporarily omitted. The discussed properties do not qualitatively depend on these.

3.2 Basic stochastic differential equation and price distribution

To begin with, it is necessary to understand basic stochastic and statistical features of modeling based on the area hyperbolic sine function:⁴

$$x = \sinh^{-1}(p)$$

To this end, the same standard route as for the log transformation can be taken. We can analyze a basic model for the spot price process using the Ornstein–Uhlenbeck (OU) process:

$$dx = k(m - x) dt + \sigma dW \quad (3.2)$$

with k denoting the mean-reversion rate, m the mean-reversion level, W the standard Wiener process and σ the volatility.

By means of the Ito formula and standard relations for hyperbolic functions, we obtain the stochastic differential equation:

$$\frac{dp}{\sqrt{1+p^2}} = \left[k(m - \sinh^{-1} p) + 0.5\sigma^2 \frac{p}{\sqrt{1+p^2}} \right] dt + \sigma dW \quad (3.3)$$

This is similar to the stochastic differential equation for the log case:

$$\frac{dp}{p} = [k(m - \ln p) + 0.5\sigma^2] dt + \sigma dW$$

Both stochastic differential equations generate the same dynamics for large p . Additionally, (3.3) exhibits a linear form $dp \approx L(p) dt + \sigma dW$ for small $|p|$. There is an explicit probability distribution solution to (3.3). The stationary distribution to the OU process for x :

$$x \sim N\left(m, \frac{\sigma^2}{2k}\right) \equiv N(m, \sigma_{OU}^2)$$

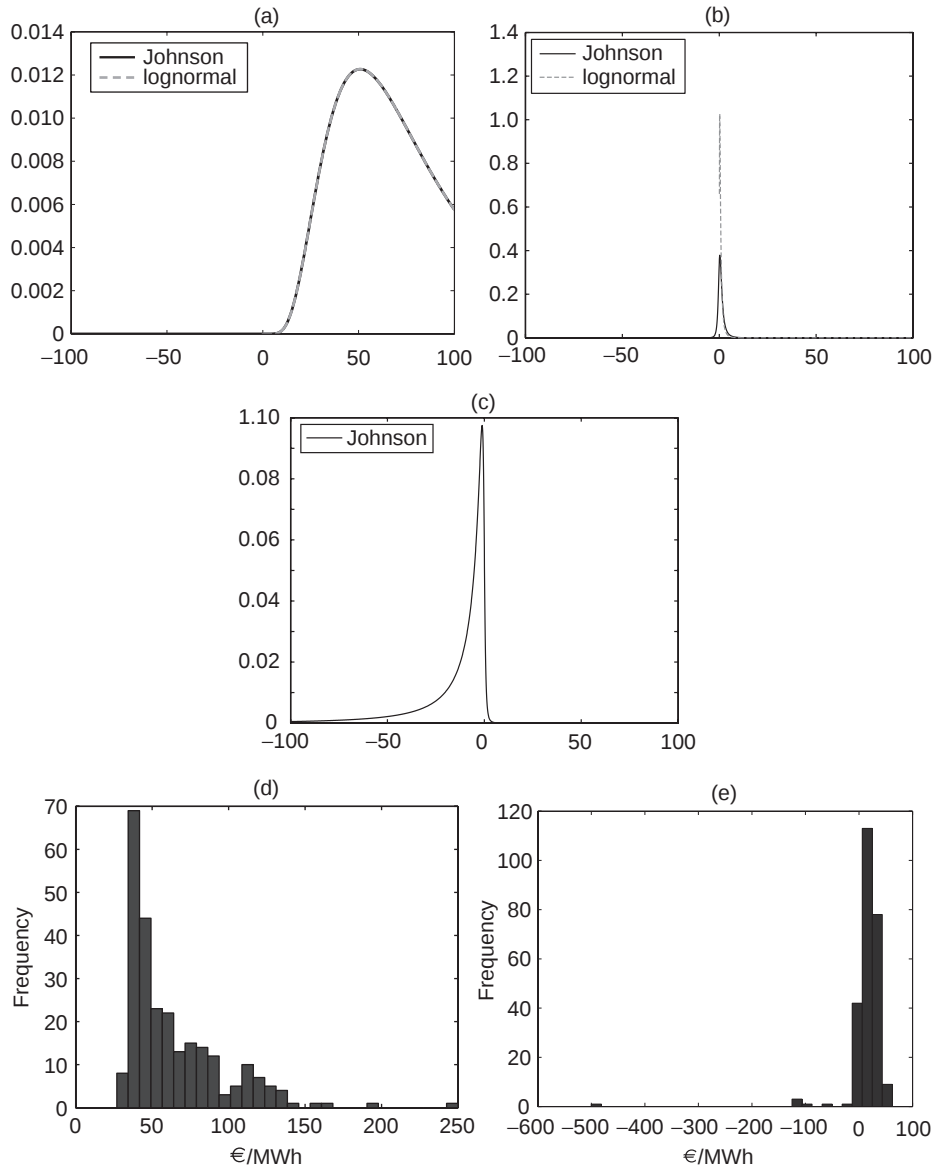
reads, for p :

$$f(p) = \frac{1}{\sigma_{OU} \sqrt{2\pi} \sqrt{1+p^2}} \exp\left(-\frac{(\sinh^{-1} p - m)^2}{2\sigma_{OU}^2}\right) \quad (3.4)$$

where f is known as the Johnson SU distribution (Johnson *et al* (1994)). It has to be pointed out that switching from the log to the hyperbolic sine transformation has not qualitatively complicated the analysis. We still obtain a closed-form solution, with the Johnson distribution replacing the lognormal distribution.

⁴ Stochastic processes involving the hyperbolic sine have been employed before to model volatility smiles. However, prices in that context are strictly positive and described by ordinary returns (see Brigo *et al* (2003) and Carr *et al* (1999)).

FIGURE 4 Qualitative comparison of empirical spot price distributions and theoretical distributions.



(a) Probability distributions with mean 80, standard deviation 40. (b) Probability distributions with mean 1, standard deviation 2. (c) Probability distributions with mean 20, standard deviation 20. (d) EEX hourly prices, working days, early evening. (e) EEX hourly prices, Sunday/holidays, deep night hours. Theoretical distributions: Johnson and lognormal with same mean and standard deviation (except for the negative mean case in part (c)).

Figure 4 on the preceding page depicts typical examples of the Johnson distribution for different means and variances, overlaid with lognormal distributions with the same mean and variance with respect to p . This is not feasible for negative means. Additionally, empirical histograms of EEX spot prices are displayed. For large (positive) mean and large variance, the Johnson and the lognormal distribution cannot be distinguished, as expected. Empirically, this case corresponds to the histogram of typical high-price hour types (working day, early evening). A right-skewed, upward-spiking (and fat tail) behavior is observed. Moving to typical lowest-price hour types (Sundays and holidays, the middle of the night), the opposite is observed, with left-skewed and downward-spiking behavior. The second case can also be (qualitatively) captured by a Johnson distribution, but not by a lognormal distribution.

4 SPOT PRICE MODELING

It is now necessary to analyze how the hyperbolic sine transformation integrates with state-of-the-art spot price models and whether the simulation results are able to reproduce the spot price histories with negative prices in a satisfactory way.

The “independent spike model” is taken as the state-of-the-art spot price model (see De Jong (2006) and De Jong and Schneider (2009)). The model produces daily spot prices by means of a three-regime-switching process, regimes for normal prices and upward and downward spikes. Below we give the model formulated as a discrete-time process.

Mean-reverting regime M: $dx_t^M = \alpha(\mu - x_{t-1}^M) + \sigma \varepsilon_t$.

Spike regimes: $x_t^S = \mu + \sum_{i=1}^{n_t+1} Z_{t,i}$.

High spike regime: $Z_{t,i} \sim N(\mu^H, \sigma^H)$, $n_t \sim \text{POI}(\lambda^H)$, $\mu^H > 0$.

Low spike regime: $Z_{t,i} \sim N(\mu^L, \sigma^L)$, $n_t \sim \text{POI}(\lambda^L)$, $\mu^L < 0$.

Markov transition matrix:

$$\Pi = \begin{bmatrix} 1 - \pi^{\text{MH}} - \pi^{\text{ML}} & \pi^{\text{MH}} & \pi^{\text{ML}} \\ \pi^{\text{HM}} & 1 - \pi^{\text{HM}} & 0 \\ \pi^{\text{LM}} & 0 & 1 - \pi^{\text{LM}} \end{bmatrix}$$

The normal price process is an OU process with mean-reversion level μ and rate α . The (log) prices in the spike regimes are drawn directly from fat-tailed distributions (Poisson compounded Gaussians). For further explanations, especially on the parameter estimation method, see De Jong (2006).

4.1 Deseasonalization

After applying the area hyperbolic sine transformation, the deterministic and trend components from the (daily) prices have to be removed. The decomposition is as follows:

$$s_t + x_t = x'_t = \sinh^{-1}(p_t)$$

with x_t being the residual stochastic component to be modeled by the De Jong model. The component s_t is further decomposed into the deterministic component d_t for day-type effects and a component y_t for the long-term trend behavior.

The day-type-specific component is captured by dummy variables $1_D(t)$, $D = \{\text{Mon}, \dots, \text{Fri}, \text{Sat/bridge days}, \text{Sun/holidays}\}$ being the day-type set.⁵

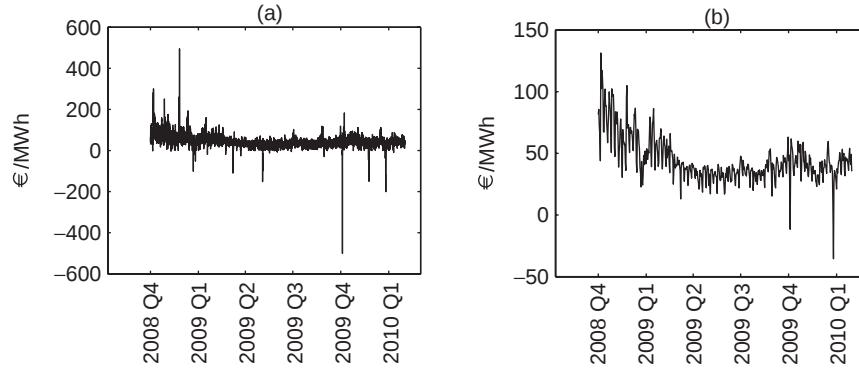
The trend or long-term component is composed of, for example, changing plant fuel prices and changing demand. In a more comprehensive approach, the component y_t could be modeled as a second, slow stochastic factor to which the fast component x_t mean-reverts (Schwartz and Smith (2000)). However, as stated previously, it is not the focus of this work to detail spot models, but, instead, to demonstrate that spot models can be brought together with the proposed hyperbolic sine price transformation seamlessly. The component y_t can be removed from the time series by means of a moving average approach (De Jong (2006)). Specifically, we employ the approach $\text{MA}(t) = x'(t') * G(t - t')$, where $G(t - t')$ is a Gaussian convolution kernel with a standard deviation of twenty days. A standard method is applied where all data points that deviate from the mean by more than three standard deviations are removed repeatedly until no more outliers are left. This is because $\text{MA}(t)$ is meant to be the mean-reverting level of the normal price process. The choice of a standard deviation of twenty days is motivated by the standard notion in power trading that a time horizon shorter than a month is short-term and spot related. Slowly moving fundamentals (for example, fuel prices and winter–summer effects) only become noticeable on a longer timescale. Note that the estimation of the trend component does not interfere with the estimation of the parameters of the De Jong model. The spot model's timescales, the half-life time of mean-reversion and the typical time the process stays in the spike regime are of the order of a few days (eg, two days).

The final step of deseasonalization, removing the aggregated component s_t , involves estimating the coefficients b $\text{MA}(t)$ and $d_i 1_D(t)$ for both subcomponents by linear regression.

4.2 The EEX case

As a first case study, the EEX hourly price history from October 2008 (the point in time where negative prices were introduced) to January 2010 are modeled. The

⁵ “Bridge day” means an isolated working day between a holiday and a weekend.

FIGURE 5 EEX (a) hourly and (b) daily price history, October 2008 to January 2010.**TABLE 1** Parameter estimates for the (daily) spot price model, EEX case.

Time-series parameters			Switch probabilities	
Normal regime	α	0.42	Normal to high	0.95%
	μ	0.02	High to normal	71.15%
	σ	0.11	Normal to low	1.93%
High-spike regime	μ_H	0.35	Low to normal	78.50%
	σ_H	0.06		
	λ_H	0.10		
Low-spike regime	μ_L	0.24		
	σ_L	0.04		
	λ_L	2.00		

modeling consists of two stages. First, the independent spike model is employed to produce scenarios of daily prices. Second, the daily prices are endowed with hourly profiles by means of a historical price sampling and rescaling method. Figure 5 shows the historical daily and hourly prices of the period. The two days that visibly stick out from the daily price series have been removed (October 4 and December 26, 2009). These days, the only ones so far at EEX with a negative price for the full day, are exceptionally strong downward spikes. It does not yet appear to be meaningful to model this kind of extreme event based on only two data points. Nevertheless, there are still many days in the truncated history where highly negative hourly prices are observed.

The parameter estimation is accomplished as described above and the determined parameters are shown in Table 1 on the facing page. Upward spiking is marginal, but downward spiking is pronounced (see the values of the normal-to-spike regime jumping probability π^{MX} and the Poisson parameters λ). This is, of course, plausible, since, as stated above, downward jumps have replaced the upward spikes in the times before the economic downturn.

4.2.1 Scaling of hourly profiles

A simulated daily price is converted into twenty-four hourly prices by drawing an appropriately similar day from history and matching it to the day's price using a scaling transformation. Before carrying out this transformation, the EEX historical days are tagged by month, day type D and spike regime. The historical spike regime characterization is provided by the parameter estimation algorithm. The algorithm calculates three likelihoods for each day, indicating by the three probabilities that this day's price is brought about by either the normal price regime or one of the two spike regimes.⁶ For a simulated day with price p_t^{sim} the spike regime is definite, so, among all historical days with matching month and day type, a historical regime probability weighted random choice is carried out. The historical hourly profile S_h^{hist} is then transformed to $S_{t,h}^{\text{sim}}$ such that average price equals the day's price, $\overline{S_{t,h}^{\text{sim}}} \equiv p_t^{\text{sim}}$. In the past, with only positive prices, a simple way to transform was to multiply each hour by a factor $C = p_t^{\text{sim}} / \overline{S_h^{\text{hist}}}$.⁷ In this way, in terms of hourly price differences, off-peak hour prices are shifted a little and peak hours prices are shifted strongly, which is realistic. However, when some hours of S_h^{hist} are negative, the transformation yields an implausible result. Imagine that a simulated daily average price p_t^{sim} is greater than the historical price p_t^{hist} . This implies, as observed in market data, that all single hours exhibit an increased price too. The factor transformation with $C > 1$ would shift positive hours upward, which is correct. However, negative hours would take on even more negative prices, which is, of course, the opposite of what is observed on a real market.

The search for a plausible new transformation has been guided by the following considerations. The factor transformation preserves lognormal distributions, which is the basic price distribution in the case of only positive prices, assuming a log transformation of the prices and a basic OU model for the price process. So, if all hours of a historical day are distributed lognormally, $\ln S_h^{\text{hist}} \sim N(m_h^{\text{hist}}, \sigma_h^{\text{hist}})$, scaling with the factor C yields new lognormal distributions $\ln S_h^{\text{sim}} \sim N(m_h^{\text{hist}} + \ln C, \sigma_h^{\text{hist}})$ with equally shifted means. This principle is transferred to the hyperbolic sine transforma-

⁶ The estimator's outputs are likelihoods that are converted into probabilities.

⁷ A similar sampling rescaling method has been employed in Culot *et al* (2006) for power spot market modeling (there were no negative power prices in Europe at that time).

tion and Johnson distribution context. We again consider a sample day. Find a δ such that:

$$\begin{aligned}\sinh^{-1} S_h^{\text{hist}} &\sim N(m_h^{\text{hist}}, \sigma_h^{\text{hist}}) \\ &\rightarrow \sinh^{-1} S_h^{\text{sim}} \\ &= \delta + \sinh^{-1} S_h^{\text{hist}} \\ &\sim N(m_h^{\text{hist}} + \delta, \sigma_h^{\text{hist}})\end{aligned}$$

and:

$$\overline{S_{t,h}^{\text{sim}}} \equiv p_t^{\text{sim}}$$

With $\sinh^{-1} \Delta \equiv \delta$ we get:

$$\begin{aligned}S_h^{\text{sim}} &= \sinh(\sinh^{-1} \Delta + \sinh^{-1} S_h^{\text{hist}}) \\ &= \Delta \cosh(\sinh^{-1} S_h^{\text{hist}}) + S_h^{\text{hist}} \cosh(\sinh^{-1} \Delta)\end{aligned}$$

by means of an addition theorem. In contrast to the lognormal case, the sine hyperbolic case cannot be explicitly solved for Δ . Instead, we simplify by noticing $|\sinh x| \cong |\cosh x|$, an approximation that is very good for small $|x|$ (for example $|x| \geq 2$). Thus, we can approximate:

$$\begin{aligned}S_h^{\text{sim}} &= \Delta \cosh(\sinh^{-1} S_h^{\text{hist}}) + S_h^{\text{hist}} \cosh(\sinh^{-1} \Delta) \\ &\approx \begin{cases} \Delta S_h^{\text{hist}} + S_h^{\text{hist}} \cosh(\sinh^{-1} \Delta) & \text{for } S_h^{\text{hist}} > 0 \\ \Delta(-S_h^{\text{hist}}) + S_h^{\text{hist}} \cosh(\sinh^{-1} \Delta) & \text{for } S_h^{\text{hist}} < 0 \\ 0 & \text{for } S_h^{\text{hist}} \approx 0 \end{cases}\end{aligned}$$

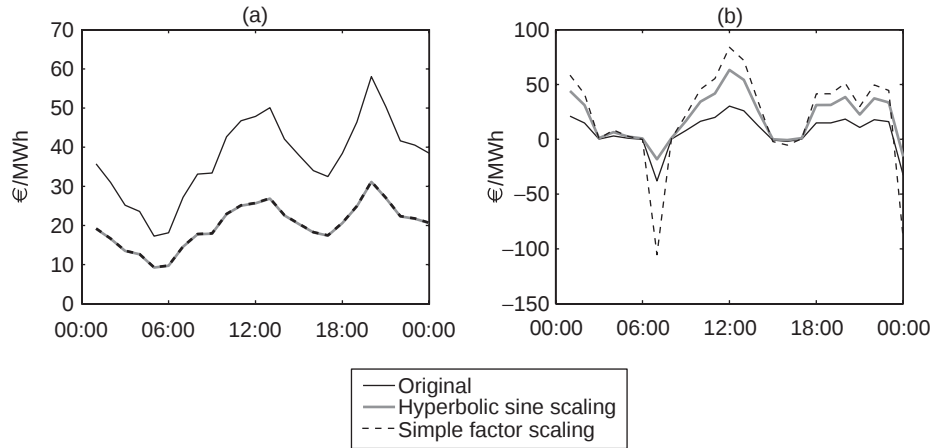
In fact, the approximation does not need to be very exact, since, for the purpose of solving for Δ , we only need price means:

$$\begin{aligned}p^{\text{sim}} &= \overline{S_h^{\text{sim}}} \\ &\approx \frac{1}{24} \sum_{S_h^{\text{hist}} > 0} S_h^{\text{hist}} (\Delta + \cosh \sinh^{-1} \Delta) + \frac{1}{24} \sum_{S_h^{\text{hist}} < 0} S_h^{\text{hist}} (-\Delta + \cosh \sinh^{-1} \Delta) \\ &= \Delta \frac{1}{24} \sum_h |S_h^{\text{hist}}| + \cosh \sinh^{-1} \Delta \frac{1}{24} \sum_h S_h^{\text{hist}}\end{aligned}$$

resulting in:

$$\overline{S_h^{\text{sim}}} \approx \Delta |\overline{S_h^{\text{hist}}}| + \cosh \sinh^{-1} \Delta \overline{S_h^{\text{hist}}}$$

which can easily be solved numerically for Δ .

FIGURE 6 Transformation of historical hourly profile.

Scaling transformations of historical hourly price profiles for EEX to new daily average prices (€20/MWh for both cases). (a) Simple factor scaled profile hidden by practically identical hyperbolic sine scaled profile. (b) Transformation of a historical hourly profile to a new profile with higher (daily) average price. The "simple factor scaling" (dashed line) produces an implausible result for the hours with a negative price.

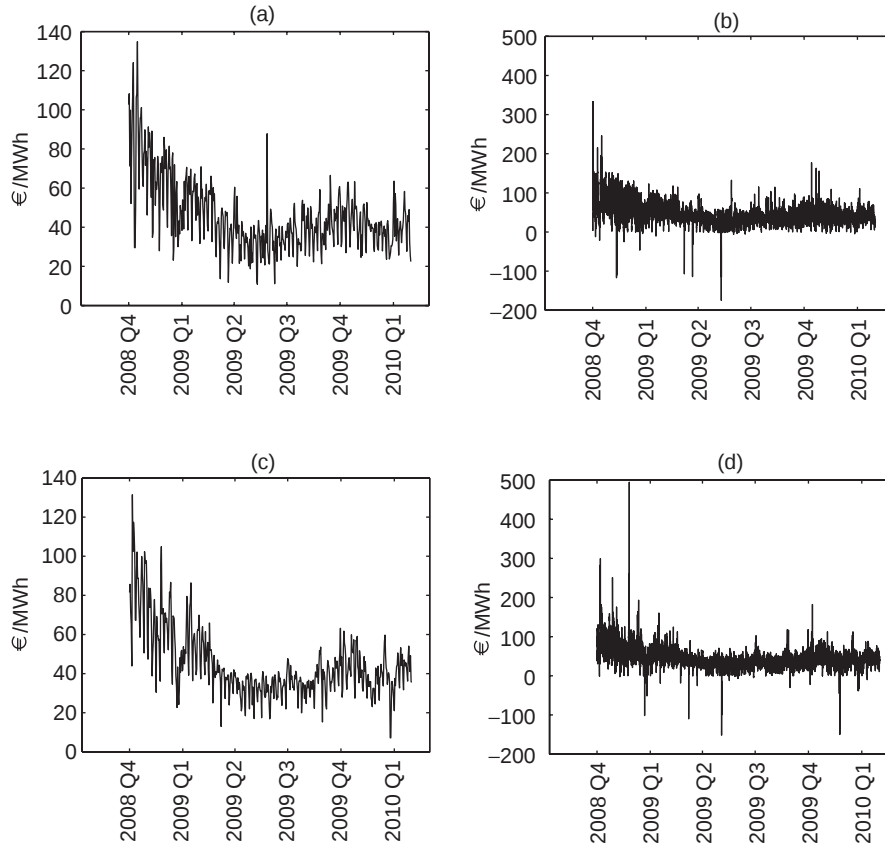
Figure 6 shows two applications of the transformation. For an hourly profile with only positive prices, the new transformation and the old factor transformation yield basically the same result (part (a)). For the profile containing negative prices, only the new transformation yields a plausible result (part (b)).

4.2.2 Simulation results

The simulated trajectories for daily and hourly prices closely resemble the historical trajectories (see Figure 7 on the next page). The goodness of fit is confirmed by comparing price statistics and their moments in Figure 8 on page 91 and Table 2 on page 92. Only the kurtosis of the historical hourly prices is not fully captured, which is a typical issue that affects virtually all power spot price models.

4.3 The ERCOT West case

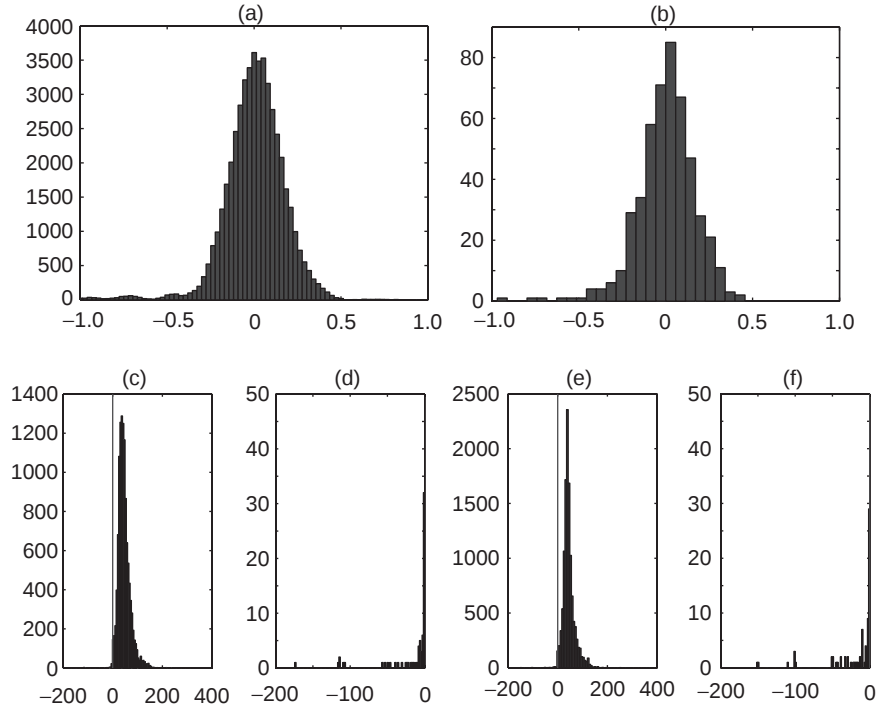
ERCOT West is a power grid zone in western Texas. This market region is specific insofar as it has quite a high share of wind power production, in conjunction with low population density and restricted transmission to other regions. The price plots in Figure 9 on page 93 show extremely volatile and spiky quarter-hourly prices,

FIGURE 7 Exemplary simulated and historical trajectories.

(a) EEX daily prices, simulated. (b) EEX hourly prices, simulated. (c) EEX daily prices. (d) EEX hourly prices.

with strong spikes occurring for positive and negative prices. Even the daily prices are remarkably volatile and spiky, with many days exhibiting completely negative average prices.

Here, the aim is to reproduce the daily prices from January 2008 to February 2010 (the history is chosen to be roughly the same time period as for EEX). Modeling is restricted to daily prices because the quarter-hourly structure is too irregular to be captured using the methods applied in this paper. Furthermore, it can be seen from the price plot that the time series is structurally changing, being significantly more spiky and negative in the first half of the period. This will be subjected to a simplification, since the overarching aim of this work is to show how to generally

FIGURE 8 Histograms of daily (x) and hourly prices.

(a) EEX deseasonalized daily prices, simulated. (b) Historical EEX deseasonalized daily prices. (c) EEX hourly prices, 1 simulation scenario. (d) Simulation, negative prices only. (e) Historical EEX hourly prices. (f) Historical, negative prices only.

integrate negative prices into stochastic spot price modeling without dealing with very specific features.

4.3.1 Extended area hyperbolic sine transformation

When applying the hyperbolic sine transformation $x'_t = \sinh^{-1}(p_t)$ as in the EEX case above, we get an unsatisfying result (see Figure 10 on page 93). Compared with the original time series, the downward spikes are strongly amplified, whereas the upward spikes are comparatively suppressed. The transformation does not preserve the characteristics of the data.⁸ As shown in Figure 11 on page 94, this is because, com-

⁸ Nevertheless, a parameter estimation for the spot price model was carried out on this data. This is, however, only successful when the model's spike regimes are endowed with a more extreme and fat-tailed distribution.

TABLE 2 Statistics of historical and simulated prices: EEX case.

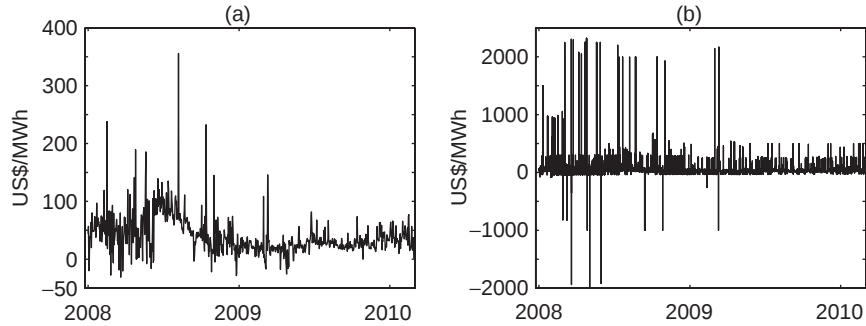
(a) Daily area hyperbolic sine prices, deseasonalized		
	Historical prices	Simulated prices
Mean	0.00	—
Standard deviation	0.18	0.18
Skewness	−1.88	−1.56
Kurtosis	13.03	11.19
(b) Daily prices		
	Historical prices	Simulated prices
Mean	44.74	44.86
Standard deviation	17.69	17.68
Skewness	1.51	1.44
Kurtosis	5.92	5.93
(c) Hourly prices		
	Historical prices	Simulated prices
Mean	44.74	44.83
Standard deviation	24.46	23.94
Skewness	1.89	1.57
Kurtosis	20.02	10.36

Simulation figures aggregated from 100 scenarios.

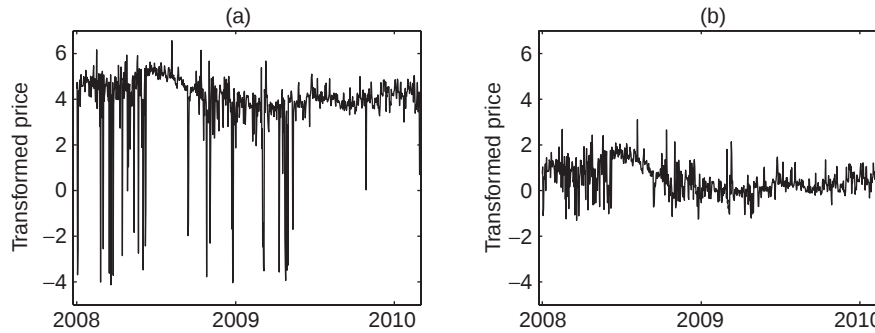
pared with range of price levels p , the $\sinh^{-1}(p)$ transformation effectively behaves in a step-like way. The main body of prices around a mean level at $p > 0$ therefore gets compressed, with the downward spikes crossing the axis being torn away. Thus, one should look for a less disruptive transformation around $p = 0$. To this end, the original form of the transformation (3.1) is reconsidered:

$$x' = \sinh^{-1} \left(\frac{p - \xi}{\lambda} \right)$$

where offset ξ and scale λ can be chosen arbitrarily. For $\xi \neq 0$ and $\lambda \neq 1$, the stochastic process characteristics stay the same. The Johnson distribution is still the basic solution to the OU process. As shown in Figure 11 on page 94, here we take

FIGURE 9 ERCOT West market clearing price history, January 2008–February 2010.

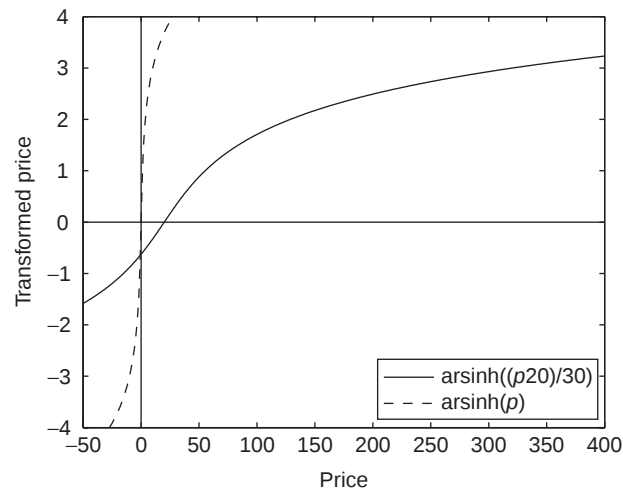
(a) ERCOT West daily prices. (b) ERCOT West quarter-hourly prices.

FIGURE 10 Applying the area hyperbolic sine transformation $x' = \sinh^{-1}((p - \xi)/\lambda)$ to ERCOT daily prices.(a) $\xi = 0, \lambda = 1$. (b) $\xi = 20, \lambda = 30$.

$\xi = 20$ and $\lambda = 30$. This shifts the turning point to the mean level of prices and widens the linear part of the transformation. Now, the transformed time series x'_t looks much more “natural” (see Figure 10). The choice of values for ξ and λ had been ad hoc, guided by the appearance of the data.⁹ In a naive fashion, the parameterization of the transformation here is somewhat similar to the idea of the Box–Cox transformation, providing an appropriate local transformation for every price level p .

⁹ Clearly, in future, there should be a mathematical method in place for determining the parameter values (see Section 6).

FIGURE 11 Depiction of area hyperbolic sine transformations as applied in Figure 10 on the preceding page.



“arsinh” stands for area hyperbolic sine.

Interestingly, Weron (2008) has already remarked, though for a setup with positive power prices only, that the log transformation does not appear to be appropriate for power prices. It produces artificial downward spikes for $p \approx 0$, although those prices are not judged to be jumping in an extraordinary way from an expert viewpoint. He then employs no transformation to the data at all (and it is therefore equivalent to the trivial linear transformation). This, however, produces the drawback of leaving spikes undampened and therefore requires special distributions for the spike regime.

The transformation elaborated in this work combines the advantages of both the log and linear approaches. The choice of ξ and λ produces a linear regime of the transformation encompassing the main, nonspiking range of prices. In the positive and negative spike price range, however, the transformation again behaves in a log-like way, allowing for the “conventional” modeling.

Why does the modified area hyperbolic sine transformation work so well? It is posited that this is because the transformation function reflects the fundamental economics of power prices. The linear range of the function coincides with the mid-merit part of the generation stack (merit order curve). In this range, we find that variable generation costs are a characteristic of “middle load” producing plants (hard coal, combined cycle gas turbine). This function increases with little curvature, is approximately linear with demand, and is not sensitive to changes in demand or supply.

In contrast, the upper generation stack, consisting of plants with high costs (eg, gas turbines), is strongly curved and sensitive to load changes. This can cause upward spikes. The lower end of the stack contains generation methods with variable costs close to zero, among others, base load plants (lignite, nuclear) and wind generation. There is no curvature, but the inflexible must-run characteristics of these generation types nevertheless make them sensitive to load changes and can lead to spot auction offers far below production costs.

4.3.2 Estimation and simulation results

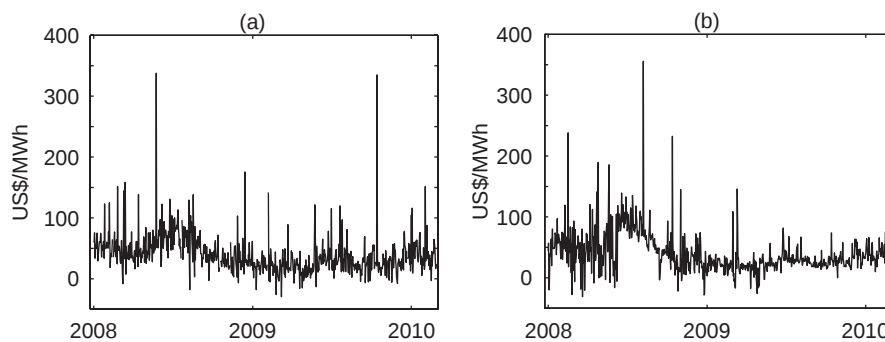
Recalling the previously described irregularities in the ERCOT price history, some modifications and simplifications have been made in parameter estimation and simulation, in contrast to the standard method for the EEX case.

The change of spike intensity over time (due to parameters changing, for example) is not accounted for. Instead, a constant parameter set is estimated from a “homogenized” history. This means that only the distribution of historical prices is fitted, discarding the original day-to-day estimation based on a Bayesian scheme. Since we know that the model price distribution is a mixture of a normal (the mean-reverting regime) and two compound normal distributions (the spike regimes), most spot price model parameters can be estimated using maximum likelihood estimation on the price distributions. One more step is needed. We need the day-to-day regime-switching probabilities for the Markov matrix, but we only know the total probability of the process being in the spike regime from the mixture-of-distributions weights. To this end, the average time that the process stays in the spike regime is estimated. All days’ prices are labeled as spikes if the normalized likelihood of belonging to the spike price distributions is greater than 0.5. The parameters are shown in Table 3 on the next page. One spike regime was discarded by the estimation. Both upward and downward spikes are captured by a single, wide distribution, which seems a more parsimonious modeling of the strongly volatile price dynamics than two distinct spike modes. The pronounced spikiness can be seen from the high value of about 20% for the probability of jumping from the normal regime into the spike regime.

Another model modification had to be applied to the simulation regarding the significant inhomogeneity of the deterministic price level (which, as for the EEX case, is estimated and then added to the simulation stochastic process). If, in the simulation, a price level combines with a spike in way that does not correspond to a historical situation (as described above, all stochastic parameters including spikes are made uniform over the complete period), some spikes with unrealistically large amplitudes are produced (approximately 0.5% of all prices). These are capped and floored to the historically observed absolute maximum and minimum. Otherwise, two or three of those events strongly distort the moments of the simulated price distribution.

TABLE 3 Parameter estimates for the (daily) spot price model, ERCOT case.

Time-series parameters			Switch probabilities	
Normal regime	α	0.59	Normal to spike	21.39%
	μ	0.03	Spike to normal	60.62%
	σ	0.32		
Spike regime	μ_S	0.00		
	σ_S	0.74		
	λ	0.50		

FIGURE 12 Exemplary simulated and historical trajectory: ERCOT case.

(a) Daily prices, simulated. (b) ERCOT West daily prices.

The simulated trajectories (see Figure 12) reveal that the “average dynamics” are matched. The effect of artificial parameter homogeneity can be observed: some spikes occur at (historically) wrong times (other trajectories match better). The moments of simulated and historical price distributions match satisfyingly (see Table 4 on the facing page), although less well than for the EEX case.

5 AN APPLICATION EXAMPLE: A SIMPLE OPTION CONTRACT ON THE SPOT PRICE

A “strip of options on the spot price” is a widespread product type in power markets. It appears in retail contracts and in wholesale/trading market business. On the one hand, these contracts serve to exchange short-term flexibility between producers and consumers, playing a role as insurance instruments. On the other hand, in the trading

TABLE 4 Statistics of historical and simulated prices: ERCOT case.

(a) Daily area hyperbolic sine prices, deseasonalized		
	Historical prices	Simulated prices
Mean	—	0.04
Standard deviation	0.53	0.52
Skewness	−0.43	0.19
Kurtosis	6.49	5.51
(b) Daily prices		
	Historical prices	Simulated prices
Mean	39.40	40.63
Standard deviation	32.40	32.83
Skewness	2.50	2.73
Kurtosis	18.39	19.78

Simulation figures aggregated from 100 scenarios.

market, they are an instrument of speculation, betting, for example, on future spot volatility. The conventional contract is a strip of call options with a defined strike. This option type is appropriate if there is a risk of “conventional spikes” (high prices) in the spot market. However, a presence of negative prices and downward spikes suggests another option type: a strip of put options on the spot price. On the one hand, this contract leaves the power producer the flexibility to produce power in low-price situations without incurring losses (as explained in Section 1, generators might sell off surplus production with large discounts). On the other hand, this contract can be an effective instrument for speculation, requiring the correct specification of spot price distributions. In this section, we focus on the second aspect and show that a trader who calculates the option premium based on a conventional lognormal specification runs the risk of significant mispricing.

The payout function for the option for maturity day T is the expected value:

$$V = E[\max[(K - p(T)), 0]]$$

given a strike K .¹⁰ In the following, it is shown that this put-option value can be calculated in closed form in the hyperbolic sine framework. Additionally, it is shown

¹⁰ For reasons of simplicity, an interest rate of zero is assumed.

that this value can differ significantly from the put option value in the conventional log price framework.

In order to make the central effect clear, the initially introduced simple OU process¹¹ is again employed, omitting a specific spike regime treatment or seasonality. As shown above, the stationary price distribution $p(T)$ is the Johnson distribution ($p \in \Re$):

$$f_{\text{SU}}(p) = \frac{1}{\lambda \sigma_{\text{SU}} \sqrt{2\pi} \sqrt{1 + ((p - \xi)/\lambda)^2}} \exp\left(-\frac{(\sinh^{-1}((p - \xi)/\lambda) - m_{\text{SU}})^2}{2\sigma_{\text{SU}}^2}\right)$$

here formulated for the full area hyperbolic sine transformation $x = \sinh^{-1}((p - \xi)/\lambda)$. When starting out with the log transformation $x = \ln p$, $p > 0$, the usual lognormal distribution:

$$f_{\text{LN}}(p) = \frac{1}{\sigma_{\text{LN}} \sqrt{2\pi} p} \exp\left(-\frac{(\ln p - m_{\text{LN}})^2}{2\sigma_{\text{LN}}^2}\right)$$

results. For V there are explicit formulas for both the log and hyperbolic cases. For the log case it is the usual expression:

$$V_{\text{LN}} = \Phi(d_{\text{LN}})$$

with Φ being the cumulative standard normal distribution function and $d_{\text{LN}} = (-m_{\text{SU}} + \ln K)/\sigma_{\text{LN}}$. For the hyperbolic case we get:

$$V_{\text{SU}} = -0.5\lambda \exp(0.5\sigma_{\text{SU}}^2) [\exp(m_{\text{SU}})\Phi(d_{\text{SU}} - \sigma_{\text{SU}}) - \exp(-m_{\text{SU}})\Phi(d_{\text{SU}} + \sigma_{\text{SU}})] \\ - \xi\Phi(d_{\text{SU}}) + K\Phi(d_{\text{SU}})$$

with:

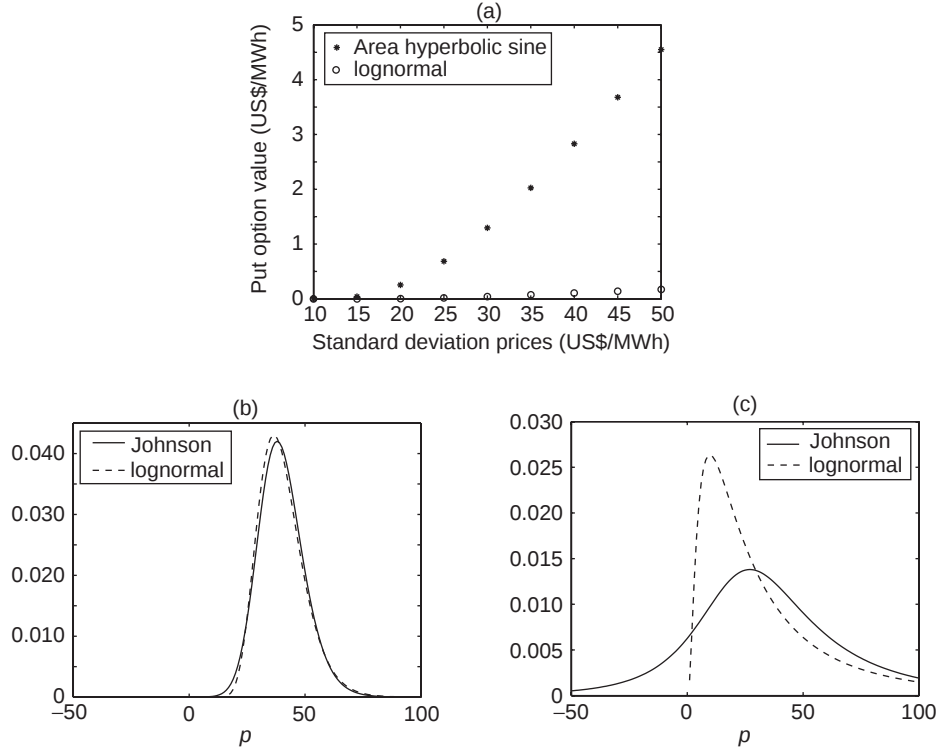
$$d_{\text{SU}} = \frac{-m_{\text{SU}} + \sinh^{-1}((K - \xi)/\lambda)}{\sigma_{\text{SU}}}$$

We assume that we know the maturity spot price $p(T)$ distribution's expected value $m = E[p(T)]$ and standard deviation σ .¹² We then can fit both the lognormal and Johnson distributions, comparing the option values based on either assumption. We have:

$$\sigma_{\text{LN}} = \sqrt{\ln\left(\frac{\sigma^2}{m^2} + 1\right)} \\ m_{\text{LN}} = \ln m - 0.5\sigma_{\text{LN}}^2$$

¹¹ A deterministic component is omitted.

¹² From the forward curve and a market volatility indicator.

FIGURE 13 An exemplary calculation.

(a) Value of the put option over standard deviation $\sigma(p(T))$ of the spot price at maturity. Johnson and lognormal distribution of $p(T)$ for two exemplary $\sigma(p(T))$. (b) Probability distributions, standard deviation price 10. (c) Probability distributions, standard deviation price 50.

and:

$$\sigma_{\text{SU}} = \sqrt{\ln \left(\sqrt{2 \frac{\sigma^2}{\lambda^2} + 2 \frac{(m - \xi)^2}{\lambda^2} + 1 + \frac{(m - \xi)^4}{\lambda^4} - \frac{(m - \xi)^2}{\lambda^2}} \right)}$$

$$m_{\text{SU}} = \sinh^{-1} \left(\frac{m - \xi}{\lambda} \exp(-0.5 \sigma_{\text{SU}}^2) \right)$$

An exemplary calculation, resembling the ERCOT market case, is carried out with $E[p(T)] = \text{US\$}40/\text{MWh}$, $K = \text{US\$}10/\text{MWh}$, $\sigma \in (\text{US\$}(5, 10, \dots, 50)/\text{MWh})$ and $\xi = 20$, $\lambda = 30$. The results are shown in Figure 13. To begin with, V_{SU} is much bigger than V_{LN} for large σ . This is obvious because $f_{\text{SU}}(p)$, unlike $f_{\text{LN}}(p)$, then comprises negative prices. However, $V_{\text{SU}} > V_{\text{LN}}$ holds for all σ , and the two valuations do not coincide for the case of a positive prices-only distribution. The

explanation comes from the form of the price distribution. The Johnson distribution attributes a bigger mass to small prices $p \approx 0$. This result indicates that the option valuation based on the hyperbolic sine framework should always be considered to avoid undervaluation, being aware of the fact that every power spot market exhibits a substantial number of prices $p \approx 0$, even if negative prices are not permitted.

As a concrete example for how values for a traded strip of put-option contracts could look, assume $\sigma = \text{US\$}20$, a contract period of one year, and a quantity of power of $e = 1 \text{ MW}$ for every hour. The total option values, then, are the products:

$$C_{\text{SU}} = V_{\text{SU}} H e \quad \text{and} \quad C_{\text{LN}} = V_{\text{LN}} H e$$

with H being the number of hours in the contract period. The difference between $C_{\text{SU}} = \text{US\$}2200$ and only $C_{\text{LN}} = \text{US\$}30$ is large and can be scaled up to much larger differences when more realistic power quantities of 50 or 100 MWh are considered.

6 CONCLUDING REMARKS

It is increasingly recognized that negative prices are an inherent feature of commodity power. Constraints on the supply side limit the flexibility of a generation facility and force the selling off of production at a discount in case of demand slumps. This is why permitting negative bids, offers and auction results in the power spot market is economically reasonable.

Several non-European power markets pioneered the use of negative prices, and the European markets EEX and Nord Pool have recently introduced negative prices. The question of whether, with the ongoing European market integration, the concept will spread to other regional markets is an interesting one. For example, a formal spot market coupling is underway between Germany and Scandinavia and between France, Belgium and the Netherlands.

It is obvious that, under these circumstances, the currently prevailing tendency to deal with the “problem” using “work-arounds” or exclusion/negligence needs to be abandoned and sound integration into the various stochastic power price modeling frameworks needs to be achieved.

The solution proposed in this paper is to replace the usual initial log transformation of prices by the area hyperbolic sine transformation. Several arguments have been provided to support this approach.

Firstly, the choice is natural, leaving the transformation for positive prices almost unchanged. The transformation function is symmetric with respect to reflection through the origin. This mirrors the finding that the price dynamic in the negative price region is analogous to the one in the positive region, with volatility basically depending on the absolute price level.

Secondly, combining the hyperbolic sine transformation with stochastic models does not significantly affect the practicability of the model compared with the log case. Interestingly, an analogue to the lognormal distribution as a theoretical “basic distribution” is found in the Johnson distribution, which is also a closed-form expression. This convenient characteristic has been exploited for the valuation of an option in Section 5.

Thirdly, the area hyperbolic sine transformation exhibits a connection to fundamentals of power prices, the generation stack and its production cost function. This is constituted, for example, by a linear regime of transformation for the moderate (normal) price regime. It has been noted before (Weron (2008)) that the log transformation does not work well in that price regime, producing artificial distortions. In this paper it was found that the area hyperbolic sine transformation (which is effectively a combination of linear and log transformation) performs well, preserving the characteristics of the data.

Summing up, it is posited that the introduction of the area hyperbolic sine transformation is the natural step for power price modeling as a response to negative prices at power spot exchanges. This is also supported by the fact that the transformation is equally well-suited for power markets with only positive prices.

There is an outstanding issue regarding negative power prices that is to be dealt with in future work. The characteristics of power markets evolve and change frequently. This is due to the complex technical nature of the fundamentals underlying power price formation. When looking at more recent EEX spot price data after early 2010 (the point in time that the work described in this paper was completed), we note that the frequency and severity of negative price occurrences is reduced, although power generation from wind has increased further. This effect, which could not be foreseen in 2009, can be attributed to a number of new factors in the market: the effect of power generators learning how to improve plant production schedules, changed regulatory directives on how to bring renewable energy to the market, and the nuclear moratorium. Today, as in 2009, there is uncertainty over what EEX spot prices are going to look like in one or two years’ time, and, specifically, uncertainty regarding the occurrence of negative prices. However, there is at least some certainty about the evolution of fundamental factors, particularly the growth of wind and photovoltaic production capacity. For the modeling of negative prices, and spot prices in general, this suggests the use of models that observe technical relations between power prices and underlying factors, especially power generation from wind. For an example of how to link fundamentals to power prices, see the approach of Howison and Coulon (2009), who relate fuel prices, demand and (conventional) power generation to power prices through a bid stack model.

REFERENCES

- Brigo, D., Mercurio, F., and Sartorelli, G. (2003). Alternative asset-price dynamics and volatility smile. *Quantitative Finance* **3**(3), 173–183.
- Carr, P., Tari, M., and Zariphopoulou, T. (1999). Closed form option valuation with smiles. Working Paper.
- Culot, M., Goffin, V., Lawford, S., de Menten, S., and Smeers, Y. (2006). An affine jump diffusion model for electricity. Seminar, Groupement de Recherche en Economie Quantitative d'Aix-Marseille.
- De Jong, C. (2006). The nature of power spikes: a regime-switch approach. *Studies in Non-Linear Dynamics and Econometrics* **10**(3), Article 3.
- De Jong, C., and Schneider, S. (2009). Cointegration between gas and power spot prices. *The Journal of Energy Markets* **2**(3), 27–46.
- Dettmer, F., and Jacob, M. (2009). Stunden, in den Strom kein "Gut" ist. *EMW* **5**, 70–72.
- Howison, S., and Coulon, M. C. (2009). Stochastic behaviour of the electricity bid stack: from fundamental drivers to power prices. *The Journal of Energy Markets* **2**(1), 29–69.
- Johnson, N. L., Kotz, S., and Balakrishnan, N. (1994). *Continuous Univariate Distributions*, Volume 1, 2nd edn. John Wiley & Sons.
- Knittel, C. R., and Roberts, M. R. (2001). An empirical examination of deregulated electricity prices. Working Paper, University of California, Energy Institute.
- Schwartz, E., and Smith, J. E. (2000). Short-term variations and long-term dynamics in commodity prices. *Management Science* **46**, 893–911.
- Sewalt, M., and De Jong, C. (2003). Negative prices in electricity markets. *Commodities Now* **June**, 74–77.
- Sprenger, S., and Laege, E. (2009). Negative Spotpreise an der EEX: Analyse und Auswirkungen auf das Risikomanagement eines Energieerzeugers. Working Paper, E.ON Energy Trading SE.
- Viehmann, J., and Sämisch, H. (2009). Windintegration bei negativen Strompreisen. *Energiewirtschaftliche Tagesfragen* **59**(11), 49–51.
- Weron, R. (2008). Heavy-tails and regime-switching in electricity prices. *Mathematical Methods of Operations Research* **69**(3), 457–473.